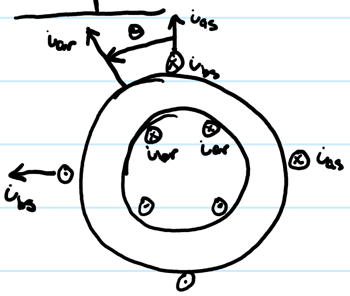
2018-12-03-1

Two phase:



$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos(\theta) & -M\sin(\theta) \\ 0 & L_s & M\sin(\theta) & M\cos(\theta) \\ M\cos(\theta) & M\sin(\theta) & L_r & 0 \\ -M\sin(\theta) & M\cos(\theta) & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}$$

$$W_m' = \frac{1}{2}\left[L_s\left(i_{as}^2 + i_{bs}^2\right) + L_r\left(i_{ar}^2 + i_{br}^2\right)\right] + M\cos(\theta)\, i_{as} i_{ar} + M\sin(\theta)\, i_{bs} i_{ar} - M\sin(\theta)\, i_{as} i_{br} + M\cos(\theta)\, i_{bs} i_{br}$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = -M\sin(\theta)\, i_{as} i_{ar} + M\cos(\theta)\, i_{bs} i_{ar} - M\cos(\theta)\, i_{as} i_{br} - M\sin(\theta)\, i_{bs} i_{br}$$

$i_{as} = I_s \cos(\omega_s t)$ $\qquad$ $i_{ar} = I_r \cos(\omega_r t)$

$i_{bs} = I_s \sin(\omega_s t)$ $\qquad$ $i_{br} = I_r \sin(\omega_r t)$

$\frac{d\theta}{dt} = \omega_m$ $\qquad$ $\theta = \omega_m t + \gamma$

$\Rightarrow T^e = -M I_s I_r \sin\left((\omega_m - \omega_s + \omega_r)t + \gamma\right)$

if $\omega_m = \omega_s - \omega_r$, then $T^e = -M I_s I_r \sin(\gamma)$

(frequency condition) $\qquad$ $P = -\omega_m M I_s I_r \sin(\gamma)$

* Constant Torque and average power!

2018-12-03-2

Voltages: $V_{as} = \frac{d\lambda_{as}}{dt}$ $\qquad \theta = \omega_m t + \gamma$

$$V_{as} = L_s \frac{di_{as}}{dt} + M\cos(\omega_m t+\gamma)\frac{di_{ar}}{dt} - M\sin(\omega_m t+\gamma)\frac{di_{br}}{dt} - \omega_m M\sin(\omega_m t+\gamma)\, i_{ar} - \omega_m M\cos(\omega_m t+\gamma)\, i_{br}$$

$$V_{as} = -\omega_s L_s I_s \sin(\omega_s t) - \omega_r M\cos(\omega_m t+\gamma)\sin(\omega_r t) I_r - \omega_r M I_r \sin(\omega_m t+\gamma)\cos(\omega_r t) - \omega_m M\sin(\omega_m t+\gamma) I_r\cos(\omega_r t)$$
$$- \omega_m M\cos(\omega_m t+\gamma) I_r \sin(\omega_r t)$$

$$V_{as} = -\omega_s L_s I_s\sin(\omega_s t) - (\omega_r+\omega_m) M I_r \sin(\omega_r t)\cos(\omega_m t+\gamma) - (\omega_r+\omega_m) M I_r\cos(\omega_r t)\sin(\omega_m t+\gamma)$$

$\omega_m = \omega_s - \omega_r$

$$V_{as} = -\omega_s L_s I_s \sin(\omega_s t) - \omega_s M I_r \sin((\omega_r+\omega_m)t+\gamma)$$

$$V_{as} = -\omega_s L_s I_s\sin(\omega_s t) - \omega_s M I_r \sin(\omega_s t+\gamma)$$

$$V_{as} = V_s\cos(\omega_s t+\alpha)$$

$$V_s\cos(\omega_s t+\alpha) = -\omega_s L_s I_s\sin(\omega_s t) - \omega_s M I_r\sin(\omega_s t+\gamma)$$
$$= \omega_s L_s I_s\sin(-\omega_s t) + \omega_s M I_r\sin(-\omega_s t-\gamma)$$
$$= \omega_s L_s I_s\cos\left(-\omega_s t-\frac{\pi}{2}\right) + \omega_s M I_r\cos\left(-\omega_s t-\left(\gamma+\frac{\pi}{2}\right)\right)$$

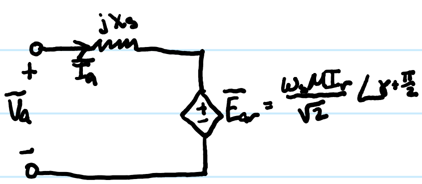
$$\boxed{V_s\cos(\omega_s t+\alpha) = \omega_s L_s I_s\cos\left(\omega_s t+\frac{\pi}{2}\right) + \omega_s M I_r\cos\left(\omega_s t+\left(\gamma+\frac{\pi}{2}\right)\right)}$$

* use $\omega_m = \omega_s$ and $\omega_r = 0$ (dc current)

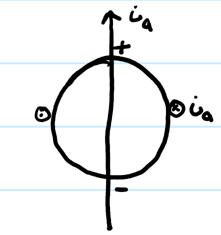
In phasor notation: $\frac{V_s}{\sqrt{2}}\angle\alpha = \frac{\omega_s L_s I_s}{\sqrt{2}}\angle\frac{\pi}{2} + \frac{\omega_s M I_r}{\sqrt{2}}\angle\gamma+\frac{\pi}{2}$

$$\frac{V_s}{\sqrt{2}}\angle\alpha = j\frac{\omega_s L_s I_s}{\sqrt{2}}\angle 0 + \frac{\omega_s M I_r}{\sqrt{2}}\angle\gamma+\frac{\pi}{2}$$

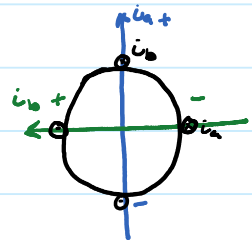
$$\overline{V_a} = jX_s\overline{I_a} + \overline{E_{ar}}$$



* Everything up to this point was a 2 pole system



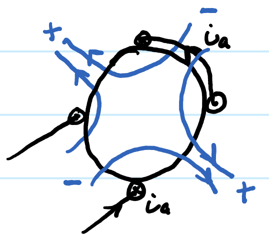
2 poles: 1+ and 1−



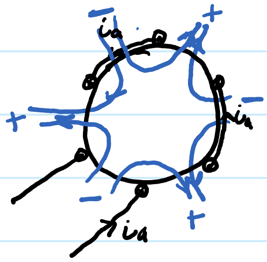
2 poles (+ and −) for each phase

* The number of poles for a system is the total number of + and − <u>per phase</u>

<u>4 pole system</u>



<u>6 pole system</u>



and so on…

Changes to equations for multi-pole systems

$p = \#$ of poles

$$\theta_e = \left(\frac{p}{2}\right)\theta_m$$

* Don't have to travel as far in θ to switch current direction

$$\omega_e = \left(\frac{p}{2}\right)\omega_m$$

θ_e = electrical angle $\qquad \theta_m$ = mechanical angle

$$\omega_m = \frac{2\pi}{60}(RPM) \qquad \omega_e = 2\pi f \Rightarrow \omega_e = 2\pi(60) \Rightarrow \omega_e = 120\pi$$

$$120\pi = \frac{2\pi}{60}(RPM) \Rightarrow \boxed{RPM = \left(\frac{2}{p}\right)3600}$$

p	RPM
2	3600
4	1800
6	1200
8	900

$P = T^e\omega_m$ (Power)

$$T^e = \frac{P}{\omega_m}$$

$$\boxed{T^e = \frac{P}{\frac{2}{p}\omega_e}}$$